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1. INTRODUCTION

Tower-based eddy covariance has been widely used to study net ecosystem-atmosphere exchanges of carbon and energy. This method traditionally requires that atmospheric flows and scalar fields are horizontally homogeneous so that the effects of advection and horizontal turbulence flux divergence components can be ignored. These requirements, however, are often not met in practice because of the surface spatial variations (surface heterogeneity) of vegetation, roughness elements, and soil moistures. In those situations, the validity of a single-point flux measurement is questionable.

The influences of surface heterogeneity on boundary layer structure and measurements are complicated because they normally occur simultaneously on a variety of scales. Blending height theory is often used to characterize the vertical extent of the influences of surface heterogeneity. Mahrt (2000) reviews different formulations of this theory. Those formulations are usually based on internal boundary layer theory (Garraatt, 1990) or Pasquill's diffusion theory (1983). Although they can provide a scaling height, they cannot predict the amplitude of the atmospheric height-dependent response to surface heterogeneity because they do not include information on the amplitude of surface heterogeneity on different scales and characteristics of the boundary layer eddies (Mahrt, 2000).

Footprint theory is also used to study the influences of surface heterogeneity. Although this theory is limited to the influence of the surface sources (or sinks) on the concentration and flux of a passive, conservative scalar, it can predict quantitative, height-dependent impacts of surface heterogeneity on scalar flux and concentration fields. The footprint is defined as the contribution of each unit element of the upwind surface area to a measured vertical flux (Schuepp et al., 1990). There are numerous footprint investigations, most of which have studied the atmospheric surface layer (e.g., Horst and Weil, 1994, Schuepp et al., 1990) using approximate analytical models and Lagrangian stochastic dispersion models. There also exist some but not many investigations on the footprint above the surface layer (e.g., Weil and Horst, 1992; Leclerc, 1997). To interpret flux measurements above the surface layer such as aircraft and tall tower measurements, one needs to consider the structure of the entire atmospheric boundary layer (ABL) and the impacts of surface

heterogeneity as a function of altitude.

The purpose of this paper is to examine the impact of surface heterogeneity on passive scalar fluxes and concentrations varies with height and with the spatial scale of heterogeneity. The footprint function in the convective boundary layer (CBL) is found by solving a three-dimensional Eulerian second-moment equation in a horizontally homogeneous atmospheric flow. The advantages of the Eulerian model used here are that it can consider the entrainment at the top of the ABL and less computation time for a larger domain compared with the Weil and Horst (1992)'s Lagrangian model and the Leclerc(1997)'s LES model, respectively.

2. METHOD

An Eulerian diffusion model with a second-order turbulence closure scheme is numerically solved to simulate the spatial distributions of mixing ratio (denoted by c for a passive scalar) and second moment quantities (e.g., $\overline{u'c'}$, $\overline{v'c'}$, $\overline{w'c'}$, where u' , v' , w' , and c' represent turbulent velocity and mixing ratio fluctuations of the scalar, respectively). The model equations are described by Stull(1988). Most of the parameterizations of the higher-order (third-order) terms and some numerical parameters in the model are the same as Mellor and Yamada(1974)'s except that a new limited length scale $k-\epsilon$ model is used to predict turbulent length scales, the turbulent kinetic energy and its dissipation rate (e.g., Apsley and Castro, 1997). The model is integrated to a quasi-steady state in the CBL with lower boundary conditions and lateral boundary conditions that are described in the following section. The horizontal grid size used in the model is 20m and the vertical grid size used is nearly logarithmically distributed in the lower part of the CBL to resolve the rapid variations of the model variables near the surface. Wind, turbulence, and potential temperature fields in the CBL are obtained through solving a one-dimensional prognostic model with the same closure scheme.

3. RESULTS AND DISCUSSIONS

3.1 Model test

To test the performance of the model, the concentration distribution of a passive scalar released from a continuous point source near the surface is simulated under convective conditions. The source is located at about 0.067h, where h is the mixing layer depth. Figure 1 presents the calculated dimensionless crosswind integrated concentration, $C_y h U / Q_0$, where C_y is the crosswind integrated concentration (kg/m^2), U is the mean horizontal wind speed (m/s), and Q_0 is the

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source strength (kg/s). The figure indicates that the centerline of the ensemble-mean plume rises rather rapidly. The centerline can reach an elevation of about $0.8h$ after a distance of the order $1.5hU/w_*$, where w_* is the convective velocity scale (m/s). These results agree well with those from the water tank experiment by Deardorff and Wills(1975), suggesting that the model can simulate the main diffusion characteristics of the CBL.

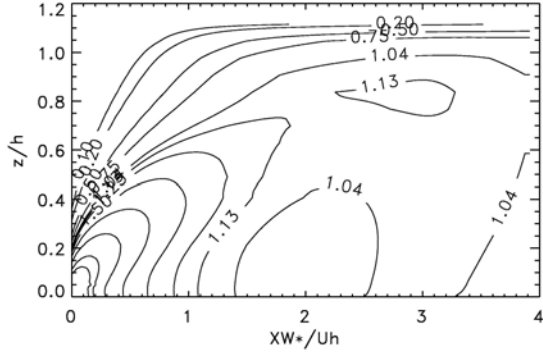


FIG. 1. Contours of calculated dimensionless crosswind integrated concentration for a point source of height, $0.067h$ under convective conditions ($h=800\text{m}$, $w_*=1.2\text{m/s}$, and $U=5\text{m/s}$).

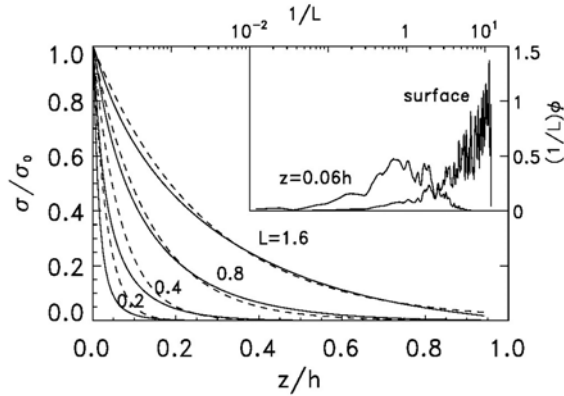


FIG. 2. Calculated (solid lines) and fitted (dotted lines) σ/σ_0 at the horizontal scales of $0.2, 0.4, 0.8,$ and $1.6 Uh/w_*$ against the height normalized by h (described by the left and bottom axes). The inset plot is spatial spectra of $w'c'$ at the surface and the height of $0.06h$ (upper right, described by the top and right axes). Note that the right axis is the product of the normalized spectral density and dimensionless spatial frequency. The top axis is the dimensionless spatial frequency (the reciprocal of a dimensionless length).

3.2 Vertical Extent Of Influences Of Surface Heterogeneity On Different Scales

Scaling arguments indicate that the influence of small scale surface heterogeneity decays more rapidly with height than that of large scale heterogeneity. This can be shown using flux conservation equation. The horizontal advection term is of the order of the production term, that is,

$$U \frac{\partial \overline{w'c'}}{\partial x} \sim \overline{w'c'} \frac{\partial U}{\partial z}, \quad (1)$$

when horizontal heterogeneity is significant. For the production term (the right-hand side), z is scaled by an energy-containing turbulence eddy length, ℓ , approximately proportional to the distance from the surface, i.e. $\ell \sim \kappa z$, where κ is a constant. If L_x is the horizontal scale of heterogeneity at the earth's surface then

$$\delta(\overline{w'c'}) \sim \frac{L_x O(\overline{w'c'})}{z}, \quad (2)$$

where δ and O denote the horizontal variation and the order of $\overline{w'c'}$, respectively. Equation (2) shows that horizontal variations in the turbulent flux decay with height (z) and increase with the scale of the heterogeneity (L_x).

To quantitatively study the effects of heterogeneous surface fluxes on flux measurements at a given height, a randomly distributed vertical scalar flux ($\overline{w'c'}$) is applied as the lower boundary condition of the model and lateral boundary conditions are set to be periodic. Then the model is integrated until the vertical flux in the simulated domain is steady (integral time is about 8 times the convective time scale, h/w_*). The fractional impact of surface heterogeneity on the turbulent flux is defined as

$$\alpha(z, L') = \frac{\sigma(z, L')}{\sigma(0, L')}, \quad (3)$$

where $\sigma(0, L')$ and $\sigma(z, L')$ are the standard deviations of the vertical fluxes at the scale L' and at the surface and the height z above the surface, respectively. Here α is a function of not only height but also the horizontal scale and can be calculated using Fourier transformation. Figure 2 presents α at the horizontal scales of $L' = 1.6, 0.8, 0.4$ and $0.2 Uh/w_*$ against the height normalized by h . These profiles are well fitted by a function (shown by broken lines in Figure 2),

$$\alpha(Z, L) = \frac{\sigma(Z, L)}{\sigma(0, L)} = \exp\left(\frac{-6.2Z}{L}\right), \quad (4)$$

where Z is the dimensionless height, z/h ; L is a dimensionless horizontal heterogeneity scale, $L = L'w_*/(Uh)$. At the height of $0.2h$, the standard deviations of the flux on the scales of $0.2, 0.4, 0.8$ and $1.6 Uh/w_*$ decay to 0, 5%, 25%, and 45% their values on the surface, respectively. It is clear that the influences of the surface heterogeneity on larger scales can survive to higher altitudes. The influence of the

heterogeneity on a 10~20km horizontal scale can exist throughout the atmospheric boundary layer because the large boundary-layer eddies that scale with the boundary layer depth (~1km) cannot mix the heterogeneity on that scale well. Figure 2 (upper right inset) also presents spatial (normalized) spectra of the flux at two heights, surface and 0.06h. The contribution of smaller scale (larger frequency) fluxes to the total variance of the flux at a given height decreases rapidly with height. Note that the right axis in the spectrum plot is the product of the frequency and the spectral density.

3.3 Footprint Function Above The Surface Layer

Vertical flux measurements for a scalar at a certain height can be regarded approximately as a weighted average of upwind surface fluxes, sometimes called the footprint function (Schuepp, et al., 1990). The averaging process is, to some degree, filtering spatial variations at scales smaller than the footprint area. Therefore, the footprint can describe the decrease of the influences of the surface heterogeneity with height.

The measured flux is an integral of the contributions from all upwind surface fluxes, i.e.,

$$F_m(x, y, z_m) = \beta \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F_0(x', y') f(x - x', y - y', z_m) dx' dy', \quad (5)$$

where F_m is the measured flux at the location (x, y, z_m) , β is the ratio of the measured flux and the surface flux, F_0 is the surface flux, and f is a relative weight function, called the footprint function that depends only on the separation between the measurement point and the location of each elemental surface flux. The summation of all f at a given height is equal to unity. Therefore, if the surface fluxes are assumed to be horizontally homogeneous and extended infinitely in space, i.e. F_0 is equal to a constant, Eq.(5) can be rewritten as,

$$F_m(x, y, z_m) = \beta F_0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x - x', y - y', z_m) dx' dy' = \beta F_0, \quad (6)$$

illustrating that β is equal to F_m/F_0 that is a linear function of height in the CBL. For measurements within the surface (or constant flux) layer, the ratio (β) is commonly assumed to be equal to unity. For the special case of a surface source with an area of $\Delta \times \Delta$ located at the origin point and an emissive flux of S_0 (F_0 is equal to S_0 only within the area and zero outside the area), substitution in (5) gives a formula for the footprint function,

$$\bar{f}(x, y, z_m) = \frac{1}{\Delta^2} \int_{-\Delta/2}^{\Delta/2} \int_{-\Delta/2}^{\Delta/2} f(x - x', y - y', z_m) dx' dy' = \frac{F_m(x, y, z_m)}{\beta S_0 \Delta^2} \quad (7)$$

where $\bar{f}(x, y, z_m)$ is the average footprint function over the area of Δ^2 . In order to find the footprint function above the surface flux layer, the vertical flux F_m and β are needed. To find F_m , the surface flux in the second-order closure model is set to zero except over a small area where it is S_0 (nonzero). At lateral boundaries, the horizontal gradients for all model variables are set to be

zero. After the vertical flux equation is integrated to a steady state, the calculated vertical flux at the point (x, y, z_m) is equivalent to the measured vertical flux $F_m(x, y, z_m)$ in Eq.(7). To find β , Eq.(7) is integrated over the entire horizontal space at a given height, resulting in

$$\beta = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F_m dx' dy' / (S_0 \Delta^2), \quad (8)$$

where the property of unit cumulative footprint over the entire horizontal space is used.

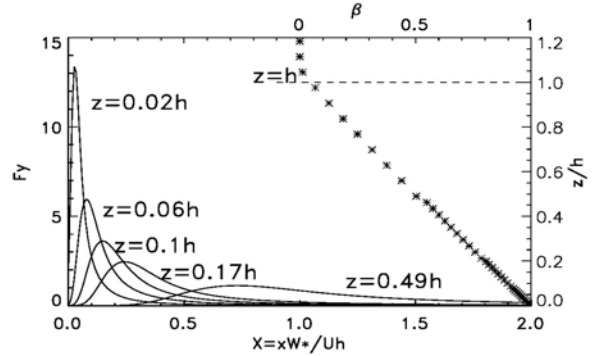


FIG. 3. Crosswind integrated footprint functions (solid lines, described by the left and bottom axes) as a function of the dimensionless distance at different heights of 0.02h, 0.06h, 0.1h, 0.17h, and 0.49h, and the vertical variation rate (β , asterisks, described by the right and top axes) of the flux versus the dimensionless height.

Figure 3 (solid lines described by the left and bottom axes) presents crosswind integrated footprint functions (F_y is defined as the integration of f from $-\infty$ to ∞ in the crosswind direction) against dimensionless upwind distances at heights of 0.02h, 0.06h, 0.1h, 0.17h, and 0.49h. It can be found that the footprint function is broadened and its peak is reduced with height, implying that the higher the altitude of the measurement, the larger the averaged area is, and hence the effects of the surface heterogeneity are reduced with height. Figure 3 (asterisks described by the right and top axes) also presents β varying with height. As shown in the figure, β varies approximately linearly with height in the form of $\beta = 1 - 0.9(z/h)$, indicating that the entrainment flux is about 10% the surface flux.

The crosswind integrated footprint function, F_y can be fitted by

$$F_y(X, Z) = \frac{3.2Z}{X^2} \exp\left(-\frac{3.2Z}{X}\right), \quad (9)$$

where X and Z are dimensionless distances. This formula is similar to that for the surface flux layer under neutral stability given by Schuepp et al. (1992) (e.g., $F_y = (Uz / (\kappa u_* x^2)) \exp(-Uz / (\kappa u_* x))$), where u_* is the friction velocity, κ is von Karman constant, U is the mean velocity) except for the modified boundary layer parameters. Eq.(9) also implies that footprint functions

above and within the surface layer have similar shapes under convective conditions.

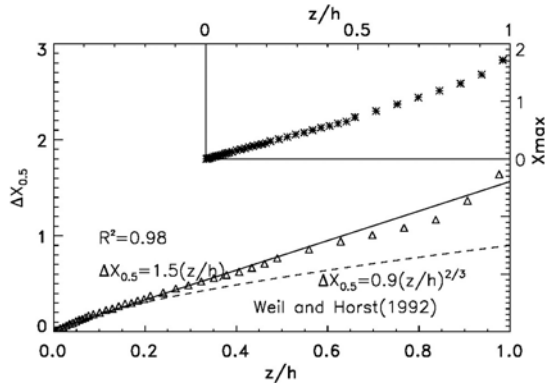


FIG. 4. Dimensionless footprint width versus the dimensionless height (described by the left and bottom axes, triangles represent calculated widths; solid line is the fitted width. Broken line is the footprint width from Weil and Horst(1992)) and the dimensionless distance of the maximum footprint function versus the dimensionless height (asterisks, described by right and top axes).

Figure 4 presents the dimensionless footprint width defined as the horizontal distance, $\Delta X_{0.5}$, between the points where the flux falls to one-half its maximum value (Weil and Horst, 1992). It can be found that the width increases with height. This dependence can probably be linked to height-dependent turbulence characteristics and the diameter of the thermals responsible for the upward transport in the CBL. The fitted curve agrees well with that given by Weil and Horst(1992) in the lower part of the CBL. The difference at the upper part may be due to the different turbulence parameterizations used. The dimensionless upwind distance of the maximum values of the footprint function against the dimensionless height is presented in the upper right inset of the figure and can be expressed as a simple function of dimensionless height, $X_{max}=1.6(z/h)$.

4. CONCLUSIONS

A modified second-order closure model is used to estimate the footprint function and study the influences of surface heterogeneity on flux measurements within the convective boundary layer. The model is compared to the water tank experiment of Deardorff and Willis (1975). The good agreement between the modeled and the experimental results suggests that the model is capable of modeling dispersion under convective conditions. The simulations also indicate that the influences of smaller scale surface heterogeneity decay more rapidly with height than those of larger scale heterogeneity, which is consistent with scaling arguments. Quantitative studies show that the influence of surface heterogeneity on the turbulent scalar flux decays exponentially with the ratio of the height and the horizontal scale of heterogeneity. The influence of

surface heterogeneity greater than 10~20Km in extent can exist throughout the boundary layer because the boundary layer eddies cannot mix such large scale heterogeneity. From the footprint perspective, the width of the footprint function increases with height, indicating that higher altitude flux measurements average over larger areas. In the lower part of the CBL, the predicted footprint width is close to that of Weil and Horst(1992), but different in the upper part perhaps because of different parameterizations and methods applied. The shape of the footprint function in the CBL is similar to that within the surface layer given by Schuepp et al. (1992) except for the modified boundary layer parameters. In the CBL, the simulations indicate that the footprint functions above and within the surface layer are controlled by CBL parameters and have similar shapes. The footprint function is broadened and its peak is reduced with height because of turbulent mixing. The quantitative results in this study need to be tested by observational data.

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